

## Appendix

### **Glossary**

from Samuel Merrill's *Making Multi-candidate Elections More Democratic*.  
Princeton N.J.: Princeton University Press, 1988. Glossary, pages 133 to 138.

*Condorcet candidate.* The Condorcet candidate in a multi-candidate elections that candidate, if one exists, who could beat each of the others in separate pairwise contests, i.e., is preferred to each of the others by a majority.

*Condorcet completion method.* Condorcet completion method is a voting system that chooses the Condorcet candidate, if one exists, and specifies a contingency rule if one does not.

*Condorcet efficiency.* The Condorcet efficiency of a voting procedure is the proportion or percentage of a class of elections (for which a Condorcet candidate exists) in which the voting system chooses the Condorcet candidate as winner.

*Correlation.* The correlation between two random variables is a statistical measure of the tendency of the two variables to vary in concert.

*Dimension of a spatial model.* The dimension of a spatial model denotes the number of coordinates designated in each point in space. Each such coordinate may be intended to represent the position of a participant (voter or candidate) on a specific issue or characteristic.

*Impartial culture.* An impartial culture is a model of an electorate in which all preference orders (for a set of candidates) are equally likely.

*Insincere voting.* A voter's ballot is insincere if his reported preference order differs from his true preference order.

*Monotonicity.* A voting system violates monotonicity if a voter can raise a candidate in the social ordering by lowering that candidate in his individual ordering.

*Multi-candidate.* A multi-candidate election is an election in which there are three or more candidates.

*Normal distribution.* The normal (or Gaussian or bell-shaped) distribution of probability specifies that probability follow the density

$$f(x) = (1/\sigma\sqrt{2\pi}) \exp[-(x - \mu)^2/2\sigma^2].$$

[Where  $\sigma$  = the standard deviation, and  $\mu$  = the mean.]

*Polarized society.* A polarized society is an electorate in which two or more (usually disparate) preference orders predominate. Such a society with exactly two dominating preference orders is called a dual culture.

*Random society.* A random society is a model for an electorate in which, for each voter, candidate utilities are drawn independently from a uniform distribution.

*Relative dispersion.* In a spatial model of voting, the relative dispersion of candidates to voters is the ratio of the standard deviation of the candidates' positions to that of the voters. [See figure 3 and 4]

*Simulation.* A simulation is an experiment run as a model of reality. The simulations in this book are computer simulations, i.e., are run on a computer using mathematical models. They are also stochastic, that is they involve input generated to follow probability distributions.

*Social utility.* The social utility of a candidate is the total (alternatively, the average) utility of the candidate over all voters.

*Social-utility efficiency.* The social utility efficiency of a voting system is the normalized ratio between the expected social utilities of the candidate selected by the system and the candidate maximizing social utility.

*Squeeze effect.* The squeeze effect refers to the reduction in electoral success of a candidate when, in a spatial model, nearby candidates draw support away from the focal candidate.

*Standard deviation.* Standard deviation is a measure of the variation of a random variable; namely, the square root of the average squared deviation of the mean.

*Strategic voting.* Strategic voting involves any decision by the voter in marking his ballot intended to improve the outcome from his point of view. In addition to insincere voting, it includes, under approval voting for example, expansion or truncation used to optimize a voter's effect on the outcome.

*Transitivity.* A voter's preference order is said to be transitive if whenever the voter prefers A over B and B over C, he also prefers A over C. A similar definition applies to a social preference ordering.

### **Criterion Definitions**

from Phillip Straffin's Topics in the Theory of Voting.

Boston: UMAP, 1980. Chapter 2.

Condorcet winner criterion: If there is an alternative  $X$  which could obtain a majority of votes in pairwise contests against every other alternative, a voting rule should choose  $X$  as the winner.

Majority criterion: If a majority of voters have an alternative  $X$  as their first choice, a voting rule should choose  $X$ .

Monotonicity criterion: If  $X$  is a winner under a voting rule, and one or more voters change their preferences in a way favorable to  $X$  (without changing the order in which they prefer any other alternatives), then  $X$  should still be a winner.

### **Definitions used in electing multi-seat legislatures**

Coalitions: A ruling coalition of legislators must continue to agree on policies or else risk a collapse of government that requires a new election. A working coalition may form simply to pass a particular bill.

Cumulative-vote plurality gives each voter as many votes as there are offices to be filled. He may give all of his votes to one candidate or spread them out among several candidates. The first strategy is always better than the latter.

Multi-seat districts:

Non-dominated strategy:

Proportional representation (PR):

Most European parliaments have used PR since early in the 20<sup>th</sup> century. Such constitutions help small parties to earn some representation — giving them a voice, but rarely power, in their legislatures.

Single-seat districts:

Single Transferable Vote (STV): In its multi-winner form, Hare's STV becomes a bit more complicated.

Weighted Votes:

Ballot

Fill a total of 5 boxes. Fill only 1 box below each number and only 1 box on each candidate's row. Fill #1 for your favorite, #5 for least favorite.

Candidates	Rank Numbers				
	1st	2nd	3rd	4th	5th

Ballot

Fill a total of 5 boxes. Fill only 1 box below each number and only 1 box on each candidate's row. Fill #1 for your favorite, #5 for least favorite.

Candidates	Rank Numbers				
	1st	2nd	3rd	4th	5th

Ballot

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Ballot

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## Worksheets for calculating election results

You can run a C-STV election any time you have a pencil and paper. It is the quickest way for a group to find its majority opinion or the leading solutions on which to focus their discussions. The limited usefulness of C-STV to small groups was discussed on page 22. Computer users, see page 28.

None or the status quo should be a candidate in any election. If it wins, the committee or chairperson may have the power to decide to put the issue aside, or to continue discussions and hold another election, or to accept the result.

Voters tend to favor the first name listed on any ballot. Multi-candidate ballots give less advantage to the top position than do ballots with only two candidate. This advantage should not go to any one of the major candidates. For a large electorate, you can print several versions of the ballot with each candidate in the first position on her share of the ballots. If you cannot afford printing variations, or if they would confuse your voters, then "None" should take the top position on the ballot list.

Use a tic-sheet like this for recording the pairwise preferences from each ballot.

Pairwise comparisons											
	None		A		B		C		D		
	voters for ROW	voters for COL.	voters for ROW	voters for COL.	voters for ROW	voters for COL.	voters for ROW	voters for COL.	voters for ROW	voters for COL.	
None	—	—									
A			—	—							
B					—	—					
C							—	—			
D									—	—	

Write the totals as printed numbers on a similar table. If one row has only ratios greater than one to one (1:1), for example, 12:7, 10:9, 11:8, and 14:5, then that row's candidate is the Condorcet winner. (The ratios in that candidate's column should each be less than 1:1, in this example 7:12, 9:10, 8:11, and 5:14.) If no Condorcet winner exists, then use a tic-sheet like the one below to total the first-place votes.

Pairwise comparisons											
	None		A		B		C		D		
	voters for ROW	voters for COL.	voters for ROW	voters for COL.	voters for ROW	voters for COL.	voters for ROW	voters for COL.	voters for ROW	voters for COL.	
None	—	—									
A			—	—							
B					—	—					
C							—	—			
D									—	—	

If no Condorcet winner exists, then use a tic-sheet to total the first-place votes.

First-Place Votes					
None	A	B	C	D	

The committee or chairperson may be given the power to choose to table a cyclical issue, or to continue debate, or to accept the result of eliminations. If the later is chosen, then find the candidate who has the fewest first-place votes. Draw lines through that candidate's row and column to eliminate her. Does any one of the remaining candidates win all comparisons? If not, eliminate the candidate who is now the weakest in first-place votes. Continue until one candidate beats each of the remaining ones.



<i>None</i>	0	<i>A</i>	III	3	<i>B</i>	II	2	<i>C</i>	<del>III</del> I	6	<i>D</i>	II	2
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The chair may choose to table a cyclical issue, to continue debate, or to eliminate. Find the candidate who has the fewest first-place votes. Draw lines through that candidate's row and column to eliminate her. Does any one of the remaining candidates win all comparisons? If not, eliminate the candidate who is now the weakest in first-place votes. Continue until one candidate beats each of the remaining ones.





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