Comparison of Voting Systems

Definitions

The oldest and most often used voting system is called *single-vote plurality*. Each voter gets one vote which he can give to one candidate. The candidate who gets the most votes, a plurality, wins. In multi-candidate races the winner often gets less than a majority, less than 50% of the votes.

The *runoff* system starts with a single-vote plurality election. The top two finishers of that election go on to a new campaign and a new one-on-one election.

Approval voting was first promoted in the 1970's. It has recently been put into use by several professional societies in the United States. It lets a voter give one vote to each candidate. Brams suggests each voter cast an approval for one of the top two candidates and as many minor candidates as he rates above that one. The candidate with the most approvals wins. Note that a majority is not required.[1]

The system of "counts" created by Jean-Charles de *Borda* [2] in 1781 gives a candidate points for each rank voted. A first-rank vote gives points equal to the number of candidates minus one. A second rank vote gives points equal to the number of candidates minus two and so on. The candidate who gets the most points wins. Duncan *Black's* 1958 rule elects the Condorcet winner if one exists. Otherwise it elects the Borda winner.

Clyde *Coombs'* 1954 alternative vote, like Hare's, eliminates candidates until one gets a majority. But it eliminates the candidate with the most last-place votes.

A. H. *Copeland's* 1950 rule gives a candidate 1 point for winning a pairwise contest against another candidate and -1 for losing. (In voting cycles, Copeland often produces ties – so it does not "complete" Condorcet.)

Charles *Dodgson* (author Lewis Carrol) proposed in 1876 to elect the Condorcet winner or, in the event of a cycle, the candidate who needs to change the fewest ballots to become the Condorcet winner.

John *Kemeny's* 1959 system determines how many rank pairs must be exchanged (flipped) on voters' ballots to make a candidate win by Condorcet's rule. The candidate who requires the fewest changes wins.

The *max-min* system elects the candidate with the smallest pairwise loss. (It is not the same as Dodgson. A candidate may lose pairwise elections to two rivals by 5% each. Her max-min score would be -5%. But she might have to change 10% of the ballots to become Dodgson's winner.)

In Samuel Merrill's 1988 *standard-score* system, voters rate candidates on a fixed scale, say 0 to 100. It then makes each voter's ratings average zero (some ratings become negative). It also "normalizes" the variation within a voter's ballot. This keeps any one voter from spreading out his ratings to influence the election more than others voters.[3]

Merrill and Straffin more fully explain most of the decision rules mentioned in this article. I shall not describe in detail other utility voting systems. The tables I have borrowed from other authors do not include any. Most are quite complicated. They often fail to elect a Condorcet winner, are easy to manipulate, and their ballots may confuse and burden voters. They do not fit majority rule's one person / one vote. Hopefully they can be adapted for groups such as legislatures seeking proportional outcomes for all parties.

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1 Niemi shows that "In the absence of dichotomous preferences, sincere approval voting need not select a candidate who has a majority of first place preferences." and "...approval voting may select a candidate whom the majority of voters prefers least." These faults are impossible under C-STV.

2 Each voting system traditionally is identified by its inventor's surname.

3 "...for each voter separately, replace her ratings r_i by their statistical standard scores, i.e.,

$Z_i = (r_i - \mu)/s$ where μ and s are the mean and standard deviation of the voter's ratings." "The standard-score system, because of the complexity of its decision rule, should be recommended only for a mathematically knowledgeable electorate." (Merrill pages 101 and 103)

Condorcet efficiency

Even though Condorcet winners can beat each of the other candidates in one-on-one elections, most voting rules do not always elect them. M-STV failed to do so in figure 2 and examples 2 and 5. Given 100 elections with no voting cycles, what percentage of the 100 Condorcet winners will each voting system elect? This number is a voting system's Condorcet efficiency.[4] To estimate the efficiency of each voting system, several political scientists have used computers to simulate groups of voters.

Table 7. Condorcet Efficiencies
in computer simulated elections with 4 candidates and 4 issues
data from Chamberlin & Cohen (1978)

Voting system	21 Voters: Impartial culture	21 Voters: Candidate Dispersion Low	21 Voters: Candidate Dispersion Medium	21 Voters: Candidate Dispersion High	1000 Voters: Impartial culture	1000 Voters: Candidate Dispersion Low	1000 Voters: Candidate Dispersion Medium	1000 Voters: Candidate Dispersion High
Coombs	93	96	98	99	91	81	99	99
Borda	86	83	83	92	89	85	86	97
Hare	92	72	75	90	92	32	60	84
Plurality	69	59	53	77	69	27	33	70

Table 8. Condorcet Efficiencies
in computer simulated elections with 5 candidates and 1000 voters
from Merrill, page 24

Voting system	Random society	Spatial model, Dispersion = 1.0, C = 0.5, D = 2	D = 4	Dispersion = 1.0, C = 0.0, D = 2	D = 4	Dispersion = 0.5, C = 0.5, D = 2	D = 4	Dispersion = 0.5, C = 0.0, D = 2	D = 4
Plurality	60	57	67	61	81	21	28	27	42
Runoff	82	80	87	79	96	31	44	39	62
Hare [M-STV]	88	78	86	83	97	34	50	38	72
Approval	67	74	78	81	84	73	76	75	82
Borda	85	86	89	89	92	84	87	86	88
Coombs	90	97	97	95	97	90	91	90	94
Black (Con)	100	100	100	100	100	100	100	100	100
Utility maximizer	78	83	88	88	90	80	85	83	86
% [of elections with] Condorcet winners[5]	76	99+	99	99+	99+	98	98	98	99

Spatial model refers to simulations with a bell-curved distribution of voters on each issue. A *dispersion* of 1.0 (or medium) means the average distance between candidates' opinions is as wide as the average distance between voters' opinions; 0.5 means the candidates tend to be more moderate than the voters. (See figures 3 and 4.) The latter corresponds to the assumption that most candidates seek the large group of voters in the middle of the bell curve (See figure 1). Low dispersion = 0.4 and high = 1.5.

C = 0.5 means there is some relationship between a voter's position on one issue and his position on others; C = 0.0 means there is no correspondence between issues.

D is the number of issues simulated.

Plurality has the worst scores. Runoff and M-STV also do poorly in some situations. Often M-STV's flaw results from the squeeze effect, that was shown in Figure 2. The Concorcet-completion

4 Merrill coined this term and defined it. "The Condorcet efficiency of a voting procedure is the proportion or percentage of a class of elections (for which a Condorcet candidate exists) in which the voting system chooses the Condorcet candidate as winner." (Merrill : Glossary)

5 In simulated elections, Merrill found the frequency of cycles ranged from 47.5% for elections with 10 candidates and 25 voters randomly distributed on issues, to less than 1% with 5 candidates and 200 voters normally distributed in a bell-shaped curve. This distribution simply means most voters are moderates. (pages 20, 24) Chamberlin and found similar percentages for their simulation assumptions. On ballots from actual elections of the American Psychology Association, Chamberlin, Cohen, and Coombs found Condorcet winners, therefore no cycles, in 5 out of 5 elections. Those were five-candidate races using rank-order ballots.

rules by Black, Copeland, Dodgson, and Kemeny have Condorcet efficiencies of 100% like C-STV. But manipulation of those rules can hide Condorcet winners, as we saw in Example 6. **C-STV's resistance to manipulation is key to its high Condorcet efficiency in real life**.

Merrill explores Condorcet efficiencies in more complex situations too. C-STV's Condorcet efficiencies in complex environments remain 100% by definition. It cannot drop in a polarized society as M-STV's does. It will tend to pick the most central candidate. (Please see figure 8 on page 27.) Nor could C-STV's efficiency rise (above 100%) with rising voter uncertainty about candidates' positions on issues. M-STV's efficiency does rise as voter uncertainty rises, (Merrill, page 39) but it remains lower than C-STV's. The efficiency of M-STV and other non-Condorcet rules drops as the number of candidates increases. Obviously, the elections in which M-STV picks the Condorcet winner are a subset of those in which C-STV does.

Surveys and actual elections reveal some randomness, some clusters of like-minded voters and some agreement on the candidates' relative positions left to right. A mixture of random and a spatial models roughly resembles these actual patterns. But just as random and spatial models lead to different results, so the actual data differs from both of them. Tideman reportedly foundthat even plurality picked the Condorcet winner in 95% of three-candidate elections. He used survey data to simulate rank-order ballots. (Merrill, page 70) This does not recommend plurality since its efficiency drops as the number of contestants rises and all other systems scored higher. Chamberlin and Featherston simulated ballots to resemble the clustering and distribution they found in the APA electorate. The simulated ballots [see Table 9] So the pattern of opinion dispersion and clusters effects Condorcet efficiencies. But the relative standing of the voting systems does not change.

Condorcet efficiency, the ability to choose the Condorcet winners in elections which have them, has great importance because they tend to be the median candidates and a happy result for the greatest *number of voters*. This is not necessarily the greatest *total happiness* as utility voting systems attempt to define it.

Utility efficiency

The major competitor to Condorcet efficiency is utility efficiency. It attempts to measure how likely a voting system is to elect the candidate with supporters who feel strongly and opponents who don't much care.[6] Many people are skeptical about trying to compare utilities *inter*-personally; so Condorcet efficiency remains the most widely accepted measure.

6 Researchers attempt to make utility measure the "distance" between a candidate and a voter on an issue. They average the scores for all issues to determine the expected utility value of the candidate for that voter. The candidate's averaged utility score for all voters is said to be her social utility to the electorate. The highest candidate scores from a series of elections are averaged to find the highest average possible for the social utilities from those elections. Then the social utility scores of winners under a voting rule are averaged and compared with the highest possible to give the rule's utility efficiency as a percentage of the highest possible. Researchers subtract a large number of utility points, equal to the score of a randomly selected candidate, from both the utility maximizer and the voting system's winners. The size of each score is reduced. But the difference between their scores remains the same. So the difference is now a larger percentage of a score. This exagerates the differences between voting systems on utility efficiency. You must decide whether such exageration helps you see the differences or misleads your understanding of these differences.

There are several different conceptions of "distance" (Bordley; Merrill page 42), and no standard unit to measure interpersonal utility for all types of issues. For these reasons, many people are skeptical about the meaning, comparison, and statistical manipulation of interpersonal utilities.

Table 18. Utility Efficiencies
in computer simulated elections with 5 candidates and 1000 voters.
from Merrill, page 35

Voting system	Random society	Spatial model, Dispersion = 1.0, C = 0.5, D = 2	D = 4	C = 0.0, D = 2	D = 4	Dispersion = 0.5, C = 0.5, D = 2	D = 4	C = 0.0, D = 2	D = 4
Plurality	70	64	75	74	93	-1	0	22	52
Runoff	81	86	92	88	98	28	47	48	75
Hare (M-STV)	82	88	92	91	98	40	59	52	82
Approval	90	96	96	97	98	96	96	95	98
Borda	95	98	98	98	99	97	97	96	99
Coombs	87	96	96	96	98	92	92	92	94
Black (Con)	93	97	98	98	99	96	97	96	98

Utility efficiency estimate for C-STV

Merrill concludes his chapter on utility efficiency saying that :

"The candidate with the maximum social utility is no more likely to be the Condorcet candidate than is the candidate selected by many if not most of the systems studied. That is to say, the Condorcet criterion and the criterion of maximizing social utility are in fact very different. [Please see figure 7 on page 25.]

"Looked at from the other side of the coin, however, one sees that the Condorcet candidate generally has high social utility, although she may not have the highest of all candidates. This can be seen by comparing the social-utility efficiencies of the Black and Borda systems. The two systems differ only when there is a Condorcet candidate; [Black chooses the Condorcet candidate when there is one] the fact that the former has almost as high an efficiency as the latter indicates that the Condorcet candidate has relatively high social utility, although not as high as the Borda winner even when a Condorcet candidate exists." (Merrill, page 37)

Whenever the two criteria indicate different winners, the Condorcet winner would beat the utility winner in a one on one election.

The problem with all utility voting systems is that a minority of voters can claim on their ballots that their candidate has a much higher utility value for them than any other candidate. With this claim they may be able to "steal" the election from acomplacent majority.

For spatial model simulations I estimate C-STV's social utility efficiency will be between 95% and 97%.[7] In the case where C-STV does worst, a random society, I estimate C-STV's social utility efficiency at 90%.[8] This is as high as Merrill's simulations of utility efficiency for approval voting – a voting system based on measuring social utilities. Bordley's graphs, from simulations based on somewhat different assumptions than Merrill's, show Copeland's Condorcet-completion method is usually a little less efficient than Borda but better than approval voting. Table 9 shows Merrill found the same relationship between Borda, Black and approval. Thus the Condorcet-completion rules all have very high utility efficiencies.

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7 If Black's 97% average utility efficiency results from 1 or 2% of elections (those with voting cycles) at (multiplied by) Borda's 98% utility efficiency, plus 98 or 99% of elections at Condorcet's (?)% utility efficiency , then (?) % = 96 to 98%. Then C-STV's utility efficiency = 98 or 99% of elections times 96 to 98% efficiency, plus 1 or 2% of elections times the (40 to 98 %) efficiency of Hare = a 95 to 98% utility efficiency for C-STV.

8 If Black's 93% efficiency results from 24% of elections (those with voting cycles) at Borda's 95% utility efficiency, plus 76% of elections at Condorcet's (?)% utility efficiency , then (?) = 92%. C-STV's utility efficiency = 76% of elections at 92% efficiency, + 24% at 82% = 90% utility efficiency. (I have assumed that the presence of a voting cycle does not effect Borda's efficiency in random society.)

Distribution of Winners

Chamberlin and Cohen's 1978 spatial-model simulations showed Condorcet picked the candidate "nearest" the "center of the electorate" 87% of the time. I think this suggests a political measure of political outcomes — in contrast to the economic measure of utility. To measure the dispersions of voters and candidates and the distributions of winners and budget allocations assumes that each citizen has an equal right not only to vote but to be represented and to live under government programs compatible with the citizen's philosophy. A system that produced proportional outcomes would reduce majority domination of minorites and so make empire building unattractive. The majority would lose some of its autonomy for every increase in territory.

Table 10. Nearness to the Center of the Theoretical Electorate
4 candidates with low dispersion relative to 1000 voters
from Chamberlin and Cohen (1978)

[Voting system]	Nearest [candidate]			Furthest [candidate]
Condorcet	.87	.11	.02	.00
Borda	.81	.17	.02	.00
Coombs	.75	.20	.05	.00
Hare [M-STV]	.33	.33	.29	.05
Plurality	.23	.27	.12	.38

Condorcet has the narrowest distribution. Hare has the second widest. C-STV's distribution of winners will depend on the percentage of elections with natural or manipulated voting cycles. We know that natural cycles are rare.

Perhaps Condorcet tends to elect high utility candidates because it directly compares every candidate with each of the others. Simulations by Bordley and Merrill found Condorcet's rule picked winners a bit lower in utility than Borda which uses all information in one step. Condorcet certainly beats Hare which uses only first-choice information at each of several steps. Notice that plurality tends to elect the least-favorite candidate, the one toward one edge on a scattergram. That's because she has no competition for the voters in that area of the electorate. Meanwhile other candidates split-up the first-choice votes from the electorate's center.

Manipulation Any voting system is manipulable, sometimes. That is, all decisive, non-dictatorial voting systems can be manipulated. The operant questions are 'How often is each voting system manipulable in a realistic electorate, how easy is the manipulation, and how damaging is its effect?'[9] The evidence to date suggests C-STV resists manipulation best.

Punishing the leading candidate with last-place votes

Most voting rules reward opposition voters for "punishing" the leading candidate with last-place votes. That usually hurts the leader's score, which helps the opposition's favorite candidate to win. Example 6 demonstrates this.

In contrast, punishing the leading candidate with a last-place vote cannot help the voter's first choice to win under Condorcet's rule. The voter already ranks his favorite as number 1. So an insincere ballot cannot increase the number of voters who rank his favorite, *B*, ahead of the main rival, *A*.

But the punishing vote might decrease the chance that *A* could win by Condorcet's rule, because the insincere voter might be helping another candidate, *C*, (whom he would rank below both *A* and *B* on a sincere ballot) to beat the original leader. This may make *C* win by Condorcet's rule or it may create a voting cycle. In fact, even if most voters would honestly rank *C* *last*, insincere ballots can sometimes make her a Condorcet winner. Systems which reward punishing votes are unlikely to find true Condorcet winners.I have adapted Example 6 from one Merrill (on page 66) used to prove that Condorcet-completion rules do not necessarily elect *true* Condorcet winners when voters have polling information and then vote strategically. Black's, Copeland's, Dodgson's, and Kemeny's Condorcet-completion rules all fail this real-world test.

Example 6. Punishing Vote Strategy

a) Sincere Voting

	Interest groups' ballots		
Ballot ranks	4 voters	4 voters	1 voter
1st choice	*A*	*B*	*C*
2nd	*B*	*A*	*A*
3rd	*C*	*C*	*B*

Pairwise comparisons			
A gets 5 votes to 4 against *B* etc.	*A*	*B*	*C*
***A* wins**	—	**5:4**	**8:1**
B	4:5	—	8:1
C	1:8	1:8	—

9 "...all voting systems permit manipulation, as was shown by Gibbard (1973) and Satterthwaite (1975). Thus, the practical questions for social choice theory to answer are the extent to which different systems encourage strategic calculations in voting, their effects on the nature and perceived legitimacy of the outcome, and their implications for political stability." (Merrill, page xvii)

From this pre- election poll the major voting systems produce a unanimous result:

	Candidates			
	A	*B*	*C*	
~~Agenda~~	~~√~~			
~~Plurality~~	~~4~~	~~4~~		~~tie~~
~~Runoff~~	~~√~~			
Approval 1	4	4	1	tie
Approval 2	**9**	8	1	
Black (Con)	√			
Borda	**13**	12	2	
~~Chamberlin (Con)~~	~~√~~			
Coombs	√		X	
Copeland (Con)	**2**	0	-2	
C-STV (Con)	√			
Dodgson (Con)	**0**	-1	-4	
M-STV (Hare)	√		X	
Kemeny (Con)	**0**	-1	-8	
Max-Min (Con)	**+11**	-11	-78	
Std-score (1,0,-1)	**4**	3	-7[10]	

(Con) = a Condorcet-completion system.
X = an eliminated candidate.

Now all voters know that *A* leads the race. Voters opposed to *A* can "punish" her with last-place votes to decrease her score relative to the other candidates. In Example 5 b, supporters of *B* decide to vote strategically.

b) Strategic Voting by *B's* party

	Interest groups' ballots		
Ballot ranks	4 voters	4 voters	1 voter
1st choice	*A*	*B*	*C*
2nd	*B*	*C*	*A*
3rd	*C*	*A*	*B*

Pairwise comparisons. *A* gets 5 votes to 4 against *B* etc.	*A*	*B*	*C*
A	—	5:4	4:5
B	4:5	—	8:1
C	5:4	1:8	—

A bests *B* who bests *C* who bests *A*. This voting cycle makes the Condorcet-completion rules use a second rule to decide their winners. Whether or not they are based on Condorcet, almost all rules are easily defeated by punishing votes. Our rules produce these results for the final election:

10 Standard scores of -1, 0, and 1 are used for simplicity. Of course, the system allows any value between the extremes on a continuous scale.

`

	Candidates			
	A	*B*	*C*	
Approval 1	4	4	1	tie
Approval 2	5	**8**	5	
Black (Con)		√		
Borda	9	**12**	6	
~~Chamberlin~~				~~tie?~~
Coombs	X	√	X	
Copeland (Con)	0	0	0	tie
C-STV (Con)	√		X	
Dodgson (Con)	-1	-1	-4	tie
M-STV (Hare)	√		X	
Kemeny (Con)	-2	-2	-5	tie
Max-Min % (Con)	-11	-11	-77	tie
Std-score (1,0,-1)	0	**3**	-3	

By voting strategically, *B's* supporters would win or tie the election according to most voting rules. *C's* supporters also can vote insincerely against *A*. But they would only help *B* not *C*. *A's* supporters may try to counter *B's* strategy by punishing *B*. In that case all of these rules would choose *C*, the least-liked candidate. The important point is that *A* would not need to counter *B's* strategy in this case under C-STV, or M-STV.[11] ~~If B's supporters have the will to manipulate [and to risk electing C if A's party counters] then only C-STV, and M-STV and Chamberlin are truly Condorcet efficient in this example. Only these two rules are truly Condorcet efficient in this example.~~

Frequency of manipulable elections

Punishing is simply one of the easiest ways for voters to manipulate an election.[12] How difficult is it to manipulate an election? How often can voters manipulate an election?[13] The first question requires a framework based in psychology and information science (degrees of insincerity, degrees of risk, amont or type of information needed, communication needed). The second question needs mathimatical proofs or statistical . ~~Each analysis must rest on observations and data from actual elections. Randomly generated ballots and purely mathematical analysis do not resemble accurately actual ballots and human psychology/game playing.~~ Chamberlin, Cohen, and Coombs assessed the minimum numbers of voters needed to change the winners of actual elections. They used the ballots from 5 previous elections for the presidency of the American Psychological Association.

11 Merrill states of the Hare system, "There is also no incentive, as there is under the Borda count, for a voter to move the chief rival of his favorite to the bottom of his preference order. As long as his favorite remains in the race, lower preferences are not counted. If his favorite is eliminated, there is no motivation for the voter to try to punish his former chief rival." (page 65)

12 Other manipulations include changing the sequence of preferences as the third voter did on page vii. That particular change, not voting for his first choice, is called decapitation and is most common under single-vote plurality. (Please see figure 6 on page 25.) Many other rules reward bullet voting or plunking: voting only for one's first choice.

13 This concerns the *frequency* of manipulable elections as found in *simulations* and *practise*. It does not contradict the *theoretical* proofs by Gibbard and Satterthwaite that any possible voting system is manipulable to some degree.

Tables 2 and 3. Manipulability

data from Chamberlin, Cohen, and Coombs (1984)

The total number of voters increased over the years.

Year	1976	1978	1979	1980	1981
# of voters	11560	15285	13535	15449	14223

Table 2. Minimum Coalition Sizes Necessary for Manipulations

Voting	1976		1978		1979		1980		1981	
system	U	P	U	P	U	P	U	P	U	P
Plurality	500	500	552	552	551	551	778	778	1	1
Borda	444	964	72	476	591	842	158	849	28	104
Hare	*	*	35	*	*	*	*	*	*	*
Coombs	834	1430	468	26	63	64	36	524	254	517
Approve 2	375	662	99	379	293	406	32	428	286	307
Approve 3	714	1199	454	740	373	705	868	1277	20	156
Kemeny (Con)	1312	1822	572	819	821	971	240	957	467	566
Max-min (Con)	1410	2110	575	801	783	1240	242	1006	467	566
Black (Con)	1200	1588	531	649	616	971	231	616	321	410

* = Manipulation not possible. (Con) = a Condorcet completion system.
Approve 2 = Approval votes for the voter's top 2 choices. Approve 3 = votes for his top 3 choices.
U = Uniform majority ordering P = Proportional majority ordering

Uniform and proportional majority orderings were used to fill the empty ranks of ballots on which the voters marked only their first few choices. Uniform ordering filled-up ballots randomly so as to give no net advantage to any remaining candidate. ~~The researchers state,~~ "This corresponds to the assumption that voters are indifferent to candidates whom they do not rank." Proportional ordering made the artificially-completed ballots resemble voter-completed ballots with the same top preferences. This method corresponds to an assumption that voters omitted candidates because they lacked sufficient knowledge, and that if these voters had the knowledge necessary to complete their ballots they would have done so with the same preferences as those on the similar but complete ballots. More people must conspire to manipulate a proportionally-filled ballot set than are needed to manipulate the same ballots filled uniformly.

Table 3. Minimum Coalition Sizes as a Percentage of Voters with the Incentive and Ability to Aid in Manipulation

Voting	1976		1978		1979		1980		1981	
system	U	P	U	P	U	P	U	P	U	P
Plurality	16.0	14.6	13.3	12.6	16.0	6.2	18.6	18.2	0.1	0.1
Borda	8.6	19.2	1.0	7.0	10.2	14.5	2.2	12.4	0.5	0.5
Hare	*	*	0.7	*	*	*	*	*	*	*
Coombs	16.2	28.2	6.8	0.4	1.1	1.1	0.5	7.9	4.0	8.2
Approve 2	12.6	21.3	2.7	9.5	14.2	21.2	0.8	10.5	11.0	13.1
Approve 3	21.2	33.4	11.6	23.1	14.1	26.5	27.3	39.4	0.6	4.5
Kemeny (Con)	27.7	46.4	8.1	12.1	14.1	16.7	3.5	15.0	18.9	22.4
Max-min (Con)	63.1	91.5	23.2	32.8	45.4	74.7	10.3	32.1	18.9	22.4
Black (Con)	25.4	35.2	7.6	9.6	10.6	16.7	3.3	9.6	13.0	16.2

The researchers reported:

"The most striking result is the difference between the manipulability of the Hare system and the other systems. Because the Hare system considers only 'current' first preferences, it appears to be extremely difficult to manipulate. To be successful, a coalition must usually throw enough support to losing candidates to eliminate the sincere winner (the winner when no preferences are misrepresented) at an early stage, but still leave an agreed upon candidate with sufficient first-place strength to win. This turns out to be quite difficult to do.

> "One other factor also distinguishes the Hare system from the other[s]. The strategy by which Hare can be manipulated, on the occasions when this is possible, is quite complicated in comparison with the strategies for the other methods." (Chamberlin, Cohen, and Coombs)

The authors contrast those strategies for 2 pages. As they and Merrill imply, the first preference is the rank most likely to be sincere on each ballot. ~~Voters often must change that preferrence to manipulate STV and this probably extracts a high psychological cost — more than many voters would feel comfortable with.~~ The manipulability of the three Condorcet-completion rules (Kemeny, max-min, and Black) proves that in each election a group of voters could create a voting cycle and also change a count such as the Borda used by Black's rule. ~~That will be possible for at least one party in any election.?~~ Still, page 6 shows the need to create a cycle makes C-STV even harder to manipulate than M-STV because it increases the number of voters who must be organized into a conspiracy.

Tideman's findings reportedly agree with these.(Merrill, page 70) He used data from "thermometer" surveys of voter opinions about the candidates for the 1972 and 1976 presidential nominations. It is worth noting that he found Dodgson's Condorcet-completion rule about as resistant to manipulation as Hare's (M-STV) rule. But to manipulate Dodgson's rule needs less information than STV requires about other voters' preference lists. So those who want to manipulate Dodgson can plan and coax voters into a simple strategy. ~~For now I must base this point on psychological factors. Perhaps someone will quantify it in information units to be gathered, estimated, calculated, and communicated. It may be more difficult to quantify degrees of psychological uneasiness and risk typically required for manipulating each rule.~~

Irrelevant alternatives

The winner under Condorcet's criterion cannot be changed by removing any other candidate(s), nor by introducing any less popular candidate(s). (Merrill, page 98) Political scientists would say no one can manipulate it by introducing irrelevant alternatives. Politicians rather easily can manipulate many elections under other voting systems by using this strategy. That means politicians can make the winner become a loser by introducing a candidate who is less popular than the former winner. Introducing irrelevant alternatives includes the strategy by which parties help start-up candidates on the opposite political wing to divide the opposition.

This political trick is fairly simple and common.

Table 4. Violations of Independence of Irrelevant Alternatives

Spatial model of 200 voters and 5 candidates repeated in 1,000 elections
from Merrill, page 98

Voting system	Violation %
Plurality	19
Runoff	10
Approval	9
Borda	7
Hare (M-STV)	6
Coombs	1
Black (Condorcet/Borda)	0.1

[**C-STV** (Condorcet/Hare) 0.1 estimated[14]]

[14] In 98% of Merrill's spatial-model elections there was no voting cycle, so Black's rule used the Condorcet criterion — whose results did not change due to irrelevant alternatives. In the remaining 2% of elections Black used Borda's rule — which was vulnerable to irrelevant alternatives in 7% of these elections. 2% x 7% = 0.14% which rounds to 0.1%. This was the number Merrill reported for Black, but his number came from simulation of Black. He did not inferre it from simulations of Condorcet and Borda. C-STV's score also would be zero for 98% of the elections. Add 2% of Hare's score for a total of 0.12%. These estimates assume that violations are equally common in elections with and without cycles.

Case Study. Republicans Split Northern and Southern Democrats

In the late 1950's the US House of Representatives considered a bill to increase federal funds for local schools. The Democratic Party favored the bill and had enough votes to pass it. Republicans, opposed to the bill, reasoned that if they proposed an amendment to block the funding of segregated schools, Northern Democrats would be compelled by constituents to support it. The Southern Democrats then would have no political choice but to join the Republicans in voting against the amended bill. The Northern and Southern Democrats behaved predictably and the Republicans succeeded in killing the school-funding bill.

Let's see what would have happened under different voting procedures. Here are the approximate sizes and preferences of the three voting blocks.

Example 7. Republicans, Northern & Southern Democrats

Ballot ranks	161 Northern Democrats	80 Southern Democrats	160 Republicans		Pairwise comparisons	
					Amended	*Bill*
1st	*Amended*	*Bill*	*No bill*			
2nd	*Bill*	*No bill*	*Amended*	*Bill*	80:321	—
3rd	*No bill*	*Amended*	*Bill*	*No bill*	240:161	160:241

This is a voting cycle. The amended bill beats the plain bill by 321 votes to 80 votes. The plain bill beats no bill by 241 votes to 160. And no bill beats the amended bill 240 to 161. $A > B > N > A$.

Should they pass a bill to increase funding and fight segregation? If the House votes first on funding then on desegregation both would pass; *if* the Republicans vote for the desegregation amendment they proposed. But most parliamentary procedures require voting on the amendment before the bill. So the House would pass the amendment and then defeat the bill – as actually happened. This case follows Duncan Black's rule of thumb as cited by Straffin, "...the later you bring up your favored alternative, the better chance it has of winning"(page 20) Here *Bill* which could beat *No bill* was itself beaten in the previous round by *Amended* bill.

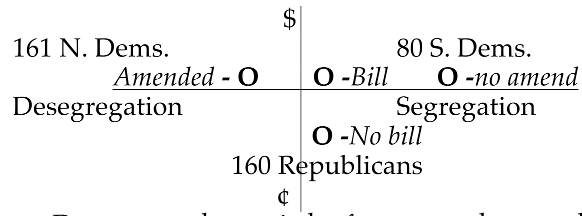
They get the same result, nothing, from the C-STV and M-STV voting systems as noted below. Keep in mind that without the amendment, the plain *Bill* would have passed by 241 Democratic votes to 160 Republican votes.

	Options			
	Amended	*Bill*	*No bill*	
~~Agenda~~			~~√~~	
~~Plurality~~	~~**161**~~	~~80~~	~~160~~	
~~Runoff~~			~~√~~	
Approve of 1	**161**	80	160	
Approve of 2	**321**	241	240	
Black (Con)	√			
Borda	**482**	321	400	
~~Chamberlin (Con)~~	~~X~~		~~√~~	
Coombs	√		X	
Copeland (Con)	0	0	0	tie
C-STV (Con)		X	√	
Dodgson (Con)	**-40**	-121	-41	
M-STV (Hare)		X	√	
Kemeny (Con)	**-40**	-121	-41	
Max-Min % (Con)	**-19.7**	-60	-20.2	

If the Northern Democrats out-number the Republicans then *Amended* bill would win by most rules. Under Copeland all options would tie. Under agenda, M-STV, and C-STV *No bill* would win. We would say C-STV was manipulated by an irrelevant alternative because the new alternative did not win, yet reversed the order of the original two options.

The Republicans might argue that they exposed the fact that some of the school funds would have gone to support racist school districts which most voters did not approve of and did not want to pay taxes for. Because of this new issue dimension, previously unconsidered, C-STV reverses its result. If the Republicans had added an amendment to set funding higher or lower than the Democrat's bill, then the C-STV result would not be reversed. *No bill* would still be defeated – by either the original *Bill* or the *Amended* bill's funding amount. The amendment certainly was not an irrelevant issue; but strictly speaking it was an irrelevant alternative.

	2 Issue Dimensions	
	Funding $	Desegregation
Yes	241	321 (*maybe*)
No	160	80



If the Republicans outnumber the Northern Democrats, that switch of one vote changes the result to *No bill* under most voting systems. If the Republicans rank the desegregation *Amended* bill last, and raise the plain *Bill* to second place, then the plain *Bill* would beat each of the other options in one-on-one contests and win under most voting systems. *No bill* could still win only under agenda and Hare.

The Republicans in this case used several manipulation techniques. First they introduced an amendment that some theorists might consider an irrelevant alternative. It created a voting cycle. Then they probably voted insincerely to punish the leading option. No one can prove insincere votes but many of these same Republicans often voted against desegregation so I doubt they sincerely preferred the *Amended* bill over the plain *Bill*.

I give this negative example of C-STV last to impress upon readers that no decisive, non-dictatorial voting system can guarantee complete resistance to manipulation in all situations. C-STV is most subject to manipulation in committee voting. Dennis Mueller writes in a section titled "Cycling", "Thus it would seem that when committees are free to amend the issues proposed, cycles must be an ever present danger." (page 64) If the amendments create a cycle, then C-STV starts to eliminate proposals. It is hard to manipulate that process, but it is possible.

Deleting less popular candidates can change the winner also. This test uses real-world ballots to measure a voting system's vulnerability to changes in the slate's minor candidates.

Table 5. Violations of Subset Rationality
data from Chamberlin, Cohen, and Coombs (1984)
Number of violations when *X* is reduced from 5 candidates to

Voting system	2 Candidates	3 Candidates	4 Candidates	Total	
Plurality	5	7	2	14	
Borda	2	2	1	5	
Hare (M-STV)	2	2	0	4	
Coombs	0	0	1	1	
Approve 2	1	17	1	19	
Approve 3	3	6	5	14	
[**Condorcet**	0	0	0	0	R.L.]

The authors note that "Violations of this subset rationality condition when a *single* candidate is omitted seem most serious..." [emphasis added] Hare was the only system with no violations when a single candidate was omitted.

There were no cycles in the real-life elections and polls studied by these authors and Tideman. So C-STV would have found Condorcet winners and each eletion's winner would not have changed due to the withdrawal of any lesser candidate(s).

Sensitivity to incomplete ballots

We may need or want to use incomplete ballots – ones with some candidates not ranked. Unfortunately all vote-counting rules will miss the most central candidate more often when voters cast incomplete ballots. Some systems will stumble due to a small percentage of bad ballots while other systems probably tolerate this problem better.

To better understand the effects of incomplete ballots, we need a study similar Chamberlin and Cohen's on the deletion of candidates, shown in Table 5. For now I shall re-use some of their published results to estimate the sensitivity to incomplete ballots for five voting systems. Tables 2a and 2b in their article showed how five election rules ranked all candidates, from winner to last-place loser. Table 2a showed their results when they filled the incomplete ballots uniformly – giving no favor to any candidate. The researchers state: "This corresponds to the assumption that voters are indifferent to candidates whom they do not rank." Table 2b gave their results when they filled the ballots proportionally – making the artificially-completed ballots resemble voter-completed ballots with the same initial preferences. This method corresponds to an assumption that voters omitted candidates because they lacked sufficient knowledge, and that if these voters had the knowledge necessary to complete their ballots they would have done so with the same preferences as those on the similar but completed ballots.

If a voting system showed many differences between those two tables, then it is very sensitive to how the incomplete ballots are filled – and probably sensitive to the use or deletion of incomplete ballots. Table 6 shows the number of differences, in winners and complete social rankings, between Chamberlin, and Cohen's Tables 2a and 2b.

Table 6. Sensitivity to Methods of Filling Incomplete Ballots
from data of Chamberlin and Cohen (1978)

	Ordering Generated by					
	Plurality	Borda	Hare (M-STV)	Coombs	Approve 2	Approve 3
Winners changed	0	1	1	1	0	0
Other positions "	0	1	1	6	1	4

Most of the systems tested by Chamberlin and Cohen sometimes picked a different winner depending on which completion method they used. A change of winners more seriously effects us than a change further down the collective ordering. So I tentatively rank the voting systems' sensitivity to incomplete ballots as: plurality, approve 2, (approve 3, Borda, Hare), and Coombs. This list seems reasonable based on how the systems select winners. Plurality always picked the same winner, runner-up and so on, no matter which completion method the researchers used. It uses only the first choice; so whatever they filled in below made no difference. Coombs eliminates the candidate with the most last-place votes; so how they filled the bottom of the ballots made a big difference.

Incomplete ballots cause no greater problem for C-STV than for most multi-candidate systems. In a later section I will argue that faulty ballots are least likely to occur under C-STV.

To sum-up this section comparing C-STV with the other voting rules: 1) C-STV probably is no more sensitive to incomplete ballots-STV has the highest possible efficiency at picking the canidate with broad support and it has a very high social utility efficiency. 3) Most importantly, C-STV resists manipulation very well *and* always elects a candidate close to the center. So in competitive political situations its winners probably will have higher Condorcet and utility efficiencies than any other voting system's. It induces the sincere ballots needed by any voting system for electing utility maximizing and Condorcet candidates and finding the greatest happiness for the greatest number of voters.