Basic Parts of C-STV

This voting system combines rank-order ballots and the Marquis de Condorcet's criterion for selecting a winner, with Thomas Hare's method of eliminating candidates until one meets the selection criterion. Joining these systems produces a descendant which has all the proven strengths of its parents but less of their most notable weaknesses.

Rank-order ballots

Rank-order ballots ask a voter to rank the candidates[1] as first choice, second choice, third and so on for as many candidates as he[2] cares to. Such ballots allow voters to choose among more than two candidates at a time. Voters do not have to deal with complex and highly-manipulable procedural rules about the order in which to vote yes or no on each option, nor tediously-repetitious and manipulable voting in run-off elections.[3] One ballot quickly and easily compares all of the candidates. The ballots contain enough information to make solid decisions with broad popular support. They express most clearly and simply the data needed for finding a candidate who meets Condorcet's criterion. Figure 1 shows part of a rank-order ballot used for national elections in Australia.

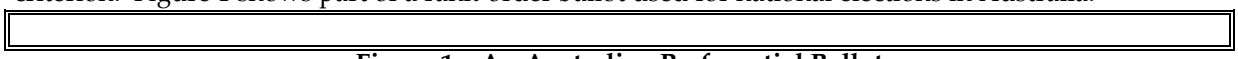

Figure 1. An Australian Preferential Ballot

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1 Candidates under C-STV can be initiatives, with and without amendments, or candidates for solitary positions such as judges, attorneys general, treasurers, and chief executives.

2 I shall use masculine pronouns for voters and feminine pronouns for candidates to help us distinguish these actors.

3 Some agenda setting is necessary even with multi-candidate voting systems. For example, take a large piece of legislation with several policy areas to be decided and several options for each policy. This leads to an array of choices such as 4 x 3 x 5...= a very large number. But most of us simply cannot study, comprehend, and rank more than 5 to 10 alternatives. So we must break-up the array into policy areas: 4 options, then 3, then 5 and so on. Multi-candidate voting systems then may let us pick each policy area's option without agenda rigging.

Condorcet's criterion

The Marquis de Condorcet's[4] criterion for picking a winner probably is respected more than any other.[5] To win, a candidate must be able to beat each of the other candidates in pairwise, one-on-one contests. To decide a pairwise contest we, in effect, electronically sort all of the rank-order ballots into two piles, one for each candidate. If a voter ranked candidate *A* above candidate *B*, then that ballot goes in *A's* pile. It does not matter whether the voter put *A* one rank or ten ahead of *B*; either way *A* wins that person's one vote in the comparison of *A* versus *B*. To the voters we might say "If you want your vote to go to candidate *A* rather than *B* when we compare *A* with *B*, then rank *A* anywhere above *B*. To do that for all pairs of candidates, just list them in the order you like them."

The Condorcet rule cannot name a winner if no candidate beats each of the others. We call this a voting paradox or cycle. Candidate *A* beats *B* who beats *C* who beats *A*. This is shown in Example 1. Recent data from computer simulations and actual elections in the U.S. suggests cycles are very rare. (Table 6 and footnote 19 will show this.) But even a rare occurrence is a critical flaw. An indecisive voting rule costs time, confusion, and the legitimacy of the ensuing government and laws. To avoid that problem several social scientists have created Condorcet-completion rules. These rules elect the Condorcet winner when one exists and use a variety of secondary rules to resolve a voting cycle when one exists.

Example 1. A Voting Cycle

Ballot ranks	Interest groups' ballots: 2 voters	2 voters	2 voters
1st choice	*A*	*B*	*C*
2nd	*B*	*C*	*A*
3rd	*C*	*A*	*B*

Pairwise comparisons
A gets 4 votes to 2 against *B* etc.

	A	*B*	*C*
A	---	**4:2**	2:4
B	2:4	---	**4:2**
C	**4:2**	2:4	---

How to read these tables and diagrams: A **bold** font marks the winning letter and its pairwise wins. An *italic* number marks a pairwise win noted later in the text. The arrows in the diagrams point from the pairwise winner to the loser in each two-candidate contest.

Pairwise comparisons
A gets 4 votes to 2 for *B* and so on.

	A		*B*	
	voters for ROW	voters for COL.	voters for ROW	voters for COL.
A	—	—	**4**	**2**
			:	
B	2	4	—	—
	:			

A Voting Cycle

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4 Born in 1743, Condorcet became an eminent mathematician, elected secretary of the Academy of Sciences and a member of the French Academy. During the French Revolution he was elected to represent Paris in the Legislative Assembly and became its secretary. He was chief author of the declaration calling for suspension of the King and summoning the National Convention – to which he offered a constitution representative of the moderate Girondins. The radical Jacobins defeated that constitution and eventually outlawed Condorcet for his forthright advocacy of political moderation. He died in prison in 1794.

5 "No criterion for evaluating voting systems appears more persuasive than that by the Marquis de Condorcet". (Merrill, page 15) "All variants of democratic theory endow a Condorcet winner with a certain degree of legitimacy, and such a mandate is no doubt a vital ingredient in the subsequent career of the winner." (Chamberlin, Cohen, and Coombs)

If the voters' ballots create a voting cycle, that moves a C-STV election into Hare's candidate-elimination process.

Alternative vote

Thomas Hare's single transferable vote is the vote-counting rule most likely to induce sincere ballots in large electorates. (Tables 2, 3, 4, and 5 will show this.) It *eliminates* the candidate(s) with the fewest first-place votes, until one candidate meets a selection criterion. On each voter's ballot, the gaps left by the eliminated candidate(s) are re-filled as the remaining candidates move up in rank. (That is shown in Example 2b.) Hare required that the winner get a majority of the (recalculated) first-place votes. In contests with more than two candidates, we usually need to eliminate some candidates before one of them can get a majority of the recalculated first-place votes. For this essay I shall call this M-STV for majority single transferable vote.[6] To the voters we might say "If you want your vote to go for *A* as long as she is a candidate, then rank her first. If *A* is dropped (for lack of first's) who would you want your vote to go to? Rank that candidate second."

Example 2. Single TransferableVote Eliminates a Candidate

a) Original ballots

Ballot ranks	Interest groups' ballots: 2 voters	1 voter	2 voters
1st choice	*A*	*B*	*C*
2nd	*B*	*A*	*B*
3rd	*C*	*C*	*A*

Pairwise comparisons: *B* gets 3 votes to 2 against *A* etc.

	A	*B*	*C*
A	—	2:3	3:2
B	*3:2*	—	*3:2*
C	2:3	2:3	—

A and *C* each get 2 first-place votes. No one gets the 3 or more first-place votes needed for a majority from the 5 voters. So M-STV requires an elimination step. *B* has the fewest first-place votes, so *B* is eliminated. *A* and *C* each move up where needed to fill gaps left by *B's* elimination.

b) After one elimination step

Ballot ranks	Interest groups' ballots: 2 voters	1 voter	2 voters
1st	*A*	*A*	*C*
2nd	*C*	*C*	*A*

Pairwise comparisons: *A* wins 3 votes to 2 against *C*.

	A	*C*
A	—	**3:2**
C	2:3	—

A gets 3 recalculated first-place votes. That is a majority so she wins under M-STV.

C-STV

If we combine Condorcet's selection criterion with Hare's elimination criterion we get C-STV. The combined voting system has a Condorcet efficiency of 100%. That means every time there is a candidate who meets Condorcet's criterion, she wins. If no candidate meets that criterion, we eliminate the weakest candidate as defined above.[7] Most importantly, there are few opportunities and great risks for voters or politicians who try to defeat a Condorcet candidate by manipulating an election. The next two sections refer to research supporting these assertions.

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6 I do not propose renaming single-winner Hare as M-STV. Likewise C-STV is only a temporary reminder of the new system's parts. Any descriptive name is apt to be misleading — so we should maintain the tradition of using inventors' surnames to label voting systems.

7 Readers can get a better feel for C-STV by ordering, say, pizza using the ballots and worksheets on page 26.

C-STV Compared with M-STV (Hare)

C-STV winners versus M-STV winners

Is a C-STV winner as "strong" a candidate as a M-STV winner? Yes. If the two winners differ, the C-STV winner is always the stronger because by definition she can beat the M-STV winner in a pairwise contest.[8]

Look again at Example 2. Candidate *A* won under M-STV. But on the original ballots, the C-STV winner, *B*, beat the M-STV winner by 3 votes to 2 votes. Straffin gives an example in which the Condorcet winner gets a 14 to 3 majority over Hare's winner. (Straffin pages 23-25)

When the two systems give different winners, C-STV's winner will always beat M-STV's.

Squeeze effect — by chance or manipulation

Example 2 can illustrate a Condorcet winner who was "squeezed-out" of a M-STV election by candidates with very similar appeals slightly to her left and right. These other candidates got more first preferences. The Condorcet-criterion winner got many second-place votes, but she got few firsts so she was eliminated before either of the two nearby candidates. Figures 2 and 5 represent this graphically.

Figure 2. A Candidate Squeeze

Candidates

Opinion positions (along 1 issue dimension)

Ballot ranks	Interest groups' numbers of voters I 6	II 2	III 2	IV 6
1st choice	*A*	*B*	*B*	*C*
2nd	*B*	*A*	*C*	*B*
3rd	*C*	*C*	*A*	*A*

Pairwise Comparisons

A loses to *B* by 6 votes to 10.

	A	*B*	*C*
A	—	6:10	8:8
B	**10:6**	—	**10:6**
C	8:8	6:10	—

As in Example 2, M-STV would eliminate *B* , although she can beat both *A* and *C*. Candidates *A* and *C* then would tie with 8 votes each.

This can occur by chance. It can also occur because politicians manipulate an election through a divide-and-conquer strategy in which they secretly help minor candidates on the opposite political wing. These candidate(s) divide the opposition into several camps, none of which can get enough votes to win. Politicians or voters can do this under M-STV, but less often than under most voting methods. Merrill describes what one must do to create a squeeze under STV voting.

> "Under the Hare [M-STV] system, manipulation on behalf of a candidate normally involves throwing some (but not too much) of the candidate's support to a pushover, who may thereby eliminate a chief rival at an early stage. Such a strategy requires a quantitative estimate of the amount of support to be shifted as well as an awkward exhortation to supporters to give first preference to another candidate in order to help their favorite. This strategy, if it is possible at all, is at once difficult to design and implausible to implement in a large electorate." (Merrill, page 75)

Several other factors make manipulations of M-STV hard. 1) Transferable-vote strategists usually must start with more first-place voters than the candidate they want to squeeze-out. 2) Strategists must know other voters' complete preference orders to know which candidate to eliminate so formerly low-ranking votes become firsts for the their nominee, or at least don't transfer to a major rival. It is not enough to know the names of the leading candidates or voters' first preferences. The number of supporters who must be encouraged to change their first preferences covers a narrow range. If strategists guess other voters' preferences incorrectly or if too many conspirators give away first preferences, then they *decrease* their chance of winning. 4) High risks of helping to elect someone *less* desirable than the candidate who would win on sincere voting also inhibit abuses of the single transferable vote: one

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8 Note that there cannot be different C-STV and M-STV winners in the same elimination step. Any M-STV winner, having an absolute majority over all other candidates combined, must also have a majority over each of the other candidates and therefore also wins by C-STV.

must squeeze-out the Condorcet-criterion winner carefully, without electing the opposite jaw of the vice. All this makes C-STV and M-STV strategies riskier than those for any other voting systems.

To the voters we might suggest "Don't reveal your preference list. Argue strongly for your favorite and against your major rival. Argue for or against the other candidates but don't let anyone know what order you rank the jokers in."

Manipulation of both C-STV and M-STV

It is possible to manipulate any voting system, sometimes. To manipulate C-STV, one must have a voting cycle. If a cycle would not occur by sincere voting and if the STV rule would not elect the the Condorcet winner, then supporters of STV's winner can manipulate the election by raising an unknown above the Condorcet winner to create a voting cycle. (The unknown then beats the former Condorcet winner who still beats the STV winner who still beats the unknown. STV's supporters rank her first and so cannot raise her any higher to beat the Condorcet winner.) Condorcet's rule can find no clear winner so the election is decided by the STV rule. C-STV will do no worse than M-STV even if manipulation of such elections succeeds. It will elect the same candidate. Thus M-STV resists manipulations better only when it inherently errs more seriously by failing to elect the one candidate whom a majority of voters support over every other candidate**.** We will estimate the opportunities for this manipulation in the next section.

If a cycle would occur without manipulation then manipulators need to put the sincere STV winner in a squeeze. But chance voting cycles are very rare and a squeeze is rarely possible and even then difficult to execute.

If by sincere voting Condorcet and STV pick the same winners, then strategists must create a voting cycle and, on the same vote, squeeze-out the former Condorcet winner. This is the most common election pattern and the hardest to manipulate. In fact manipulation is usually impossible.

Example 3. A Cycle and Squeeze

a) sincere ballots

Ballot ranks	Interest Groups			
	I 10 voters	II 3 voters	III 3 voters	IV 5 voters
1st choice	*A*	*B*	*B*	*C*
2nd	*B*	*A*	*C*	*B*
3rd	*C*	*C*	*A*	*A*

	Pairwise comparisons		
	A	*B*	*C*
A	—	10 : 11	13 : 8
B	**11 : 10**	—	**16 : 5**
C	8 : 13	5: 16	—

B beats both *A* and *C*, so C-STV would elect *B* .

b) with strategic voting by *A's* party

Ballot ranks	Interest Groups					
	Ix 2 voters	Iy 4 voters	Iz 4 voters	II 3 voters	III 3 voters	IV 5 voters
1st	*C*	*A*	*A*	*B*	*B*	*C*
2nd	*A*	*C*	*B*	*A*	*C*	*B*
3rd	*B*	*B*	*C*	*C*	*A*	*A*

	Pairwise comparisons		
	A	*B*	*C*
A	—	10 : 11	*11 : 10*
B	11:10	—	10 : 11
C	10:11	11:10	—

B beats *A* who beats *C* who now beats *B*. The 6 voters who moved *C* above *B* have created a voting cycle. No one wins by Condorcet's criterion so C-STV requires an elimination. Now that 2 of *A's* supporters have given their first preferences to *C*, *C* has more first-place votes than *B*. So we eliminate *B*.

c) after one elimination

Ballot ranks	Interest Groups					
	Ix 2 voters	Iy 4 voters	Iz 4 voters	II 3 voters	III 3 voters	IV 5 voters
1st	*C*	*A*	*A*	*A*	*C*	*C*
2nd	*A*	*C*	*C*	*C*	*A*	*A*

	Pairwise comparisons	
	A	*C*
A	—	**11 : 10**
C	10:11	—

A wins. The opportunities for this rarely occur and contain great risks. If too many of *A's* supporters, one more in this example, change their first preferences, they elect *C*, their *least* favorite candidate.

Only the 2 insincere voters in group Ix were needed to create the first-preference squeeze for M-STV. But the cycle required to manipulate C-STV needed 6 insincere voters: those 2 plus the 4 voters under Iy. Three times more conspirators were needed to manipulate C-STV than M-STV. When both C-STV and M-STV are manipulable, C-STV often resists better because it usually requires a larger number of voters and coordination of more than one tactic.

Non-monotonicity — by chance or manipulation

Occasionally, in some voting systems, a voter can hurt a candidate by raising her rank toward number one. The candidate might win on a pre-election poll, but then lose the election after some voters jump on her bandwagon and move her up to first choice. We call this system-level behavior non-monotonic. Non-monotonicity is like a bad volume control on a stereo. You turn the nob "up" and usually the volume goes up. But sometimes a bit of corrosion inside makes the volume drop lower than it was before you turned the nob. Or it is like a bad faucet: you turn the hot-water nob down and suddenly get scorched! [9] Simply in terms of logic that's a serious flaw for a decision rule. It is a theoretical possibility with any elimination process including Hare's. Voters in a non-monotonic voting system might help ⇑ a candidate by lowering ⇓ her rank as two of *A's* supporters deliberately did in Example 3. (Four other voters deliberately dropped ⇓ the leading candidate, *B*, to hurt ⇓ her – but that was monotonic.) Or they might intentionally or unintentionally hurt ⇓ a candidate by raising ⇑ her rank. In the next example, the two voters on the right will raise ⇑ the leading candidate and cause her to lose ⇓.

Example 4. Non-monotonicity in C-STV and M-STV

From Straffin, page 22

Ballot ranks	Interest groups.			
	6 voters	5 voters	4 voters	2 voters
1st choice	*A*	*C*	*B*	*B*
2nd	*B*	*A*	*C*	*A*
3rd	*C*	*B*	*A*	*C*

	Pairwise comparisons		
	A	*B*	*C*
A	—	**11:6**	8:9
B	6:11	—	12:5
C	*9:8*	5:12	—

A beats *B*, who beats *C*, who beats *A*. No one wins a majority over all nor over each of the others. Both M-STV and C-STV require elimination of the candidate with the fewest first-place votes: *C*. Then *A* wins, 11 votes to 6 votes against *B*.

Suppose the two voters on the right decided to rank *A* above *B*. *B* would then have fewer firsts than *C*. We would eliminate *B*. *C* would beat *A* by 9 votes to 8.

Notice that an elimination is necessary to give a non-monotonic result. Suppose *A* can win by M-STV without eliminations on the first poll, getting a small majority over all the other candidates

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9 Non-monotonic comes Latin words which mean not of one tone not in harmony, not in unison, or not moving together.

combined. If she gets more support later, then she will win by a larger majority. But if *A's* victory depends on facing *B* not *C*, then a shift of support from *B* to *A* may derail *A's* weak victory.

This offers another route for manipulating a M-STV or C-STV election. But most Australian political observers seem to discount the frequency of this pattern after more than 70 years of experience with Hare's elimination process. The opportunities for it are even rarer than those for the squeeze play. The improbability of both is reflected in the difficulty of manipulating Hare as will be shown in the next section.

This problem will occur even less with C-STV than with M-STV. C-STV winners need to meet only the Condorcet criterion, not the majority criterion required by Hare. We do not have to eliminate candidates until one gets a majority of the first-place votes — we usually find a Condorcet winner before that. For almost all elections we would not have to eliminate anyone. (See Table 1.) The C-STV elections which require eliminations are a subset of the M-STV elections which do so. Hence we have much less chance of causing non-monotonicity through eliminations.

Table 1. Frequencies of Majority and Condorcet Winners
in computer-simulated elections with 4 candidates
data from Chamberlin and Cohen (1978)

	Electorate sizes			
	21 voters		1000 voters	
	Majority	Condorcet	Majority	Condorcet
Impartial Culture[10]	.03	.84	.00	.85
Low Candidate Dispersion[11]	.30	.92	.00	.99
Medium Dispersion	.10	.98	.10	1.00
High Candidate Dispersion	.40	.98	.30	1.00

The last example will contrast what can happen when we have a Condorcet winner but no majority winner.

Example 5. Non-monotonicity for M-STV but not for C-STV

a) Original poll

Ballot ranks	Interest groups' ballots: 5 voters	6 voters	6 voters
1st choice	*A*	*B*	*C*
2nd	*B*	*A*	*A*
3rd	*C*	*C*	*B*

Pairwise comparisons. *A* beats *B*, 11 votes to 6.

	A	*B*	*C*
A	—	**11:6**	**11:6**
B	6:11	—	*11:6*
C	6:11	6:11	—

A beats both *B* and *C*. *A* is the Condorcet and C-STV winner. But no one gets a majority so M-STV requires elimination of the candidate with the fewest first's, *A*.

b) After the elimination of *A*

Ballot ranks	Interest groups' ballots: 5 voters	6 voters	6 voters
1st	*B*	*B*	*C*
2nd	*C*	*C*	*B*

Pairwise comparisons.

	B	*C*
B	—	**11:6**
C	6:11	

With the help of *A's* party *B* beats *C* by 11 to 6. So *B* is M-STV's winner.
But suppose 2 of *C's* supporters decide they like *B* best and so change their first choice from *C* to *B*.

10 An impartial culture has voters distributed evenly across the range of opinions on an issue.

11 This spatial-model culture has more voters in the center than on either side. A low candidate dispersion means the average candidate takes a stand closer to the center position on an issue than the average voter. This corresponds to the assumption that most candidates try to please the large group of moderate voters in the center.

c) After 2 voters shift from *C* to *B*

Ballot ranks	Interest groups' ballots: 5 voters	6 voters	4 voters	2 voters
1st	*A*	*B*	*C*	*B*
2nd	*B*	*A*	*A*	*C*
3rd	*C*	*C*	*B*	*A*

	Pairwise comparisons: *A*	*B*	*C*
A	—	9:8	11:6
B	8:9	—	13:4
C	6:11	4:13	—

A is still the Condorcet winner. But now M-STV will eliminate *C* instead of *A*.

d) After the elimination of *C*

Ballot ranks	Interest groups' ballots: 5 voters	6 voters	4 voters	2 voters
1st	*A*	*B*	*A*	*B*
2nd	*B*	*A*	*B*	*A*

	Pairwise comparisons: *A*	*B*
A	—	**9:8**
B	8:9	—

A defeats *B* by 9 to 8 (or 11 to 6 if the 2 voters keep *A* in second place and drop *C* from first to last). So M-STV's winner in the first poll was defeated due to a gain in support.

Notice that *A's* party could not produce this victory by changing their own ballots. Defeating the leading candidate by raising her rank is not a likely means of manipulation even in M-STV. It simply occurs as a random flaw in the elimination process. It is probably only a rare possibility given the patterns of preferences in actual electorates. This example and Table 1 strongly suggest that it happens even more rarely for C-STV than for M-STV.

To sum-up this section on C-STV versus M-STV: C-STV is more often monotone, and when the two systems pick different winners, C-STV's winners always beat M-STV's.